PARAMETRIC EXCITATION OF LONGITUDINAL WAVES IN A BOUNDED PLASMA BY AN HF ELECTRIC FIELD

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It is shown in [1] that volume waves are excited in an unbounded plasma situated in a uniform amplitude-modulated high-frequency (hf) electric field whose carrier frequency is much greater than the electron plasma frequency. The parametric resonance occurs because the natural oscillation frequencies of a plasma in an external hf field depend on the amplitude of this field. If the pump field is amplitude modulated at twice the natural oscillation frequency of the plasma, parametric excitation of longitudinal waves can be achieved in the system.

In this paper we consider the parametric excitation of surface modes in a bounded plasma by a modulated hf field. It is found that a parametric instability develops where the surface waves are localized near the plasma boundary if the amplitude modulation frequency is close to twice that of the surface modes. The rest of the plasma remains stable. It is thus possible to use a modulated hf field to produce heating of surface plasmons.

We consider a uniform plasma (z > 0) bounded by a medium with dielectric permittivity (ε_0 (z < 0), and suppose that the hf field is

$$\mathbf{E}(t) = \mathbf{E}_0 \sin \omega_0 t + \mathbf{E}_1 \sin \omega_1 t + \mathbf{E}_2 \sin \omega_2 t$$
$$\omega_1 = \omega + \Omega, \quad \omega_2 = \omega - \Omega, \quad \Omega \ll \omega_0, \quad \omega_0 \gg \omega_{Le} \tag{1}$$

It can be shown by a method similar to that developed in [1] that the Fourier components of the field potential of the longitudinal waves at the plasma boundary satisfy the equation

$$\Phi(z=0, \omega + n\Omega, \mathbf{k}_{\parallel})\{1 + \Delta\varepsilon_i(\omega + n\Omega)\} + \sum_{m,\, s=-\infty}^{\infty} A_{m-n} A_{m-s}\, \Delta\varepsilon_e(\omega + m\Omega)\, \Phi(z=0, \omega + s\Omega, \mathbf{k}_{\parallel}) = 0$$
$$A_n = A_{-n} = \sum_{m=-\infty}^{\infty} J_{n-2m}(a_0) J_{n+m}(a_1) J_m(a_0) \tag{2}$$
$$\Delta\varepsilon_\beta(\omega) = \delta\varepsilon_\beta'(\omega)(1 + \varepsilon_0)^{-1}; \quad a_j = e\mathbf{E}_j \mathbf{k}_{\parallel}/m_e \omega_j^2 = \mathbf{k}_{\parallel} \mathbf{r}_{Ej}; \ \beta = e, i$$

Here $\delta\varepsilon_\beta(\omega)$ is the longitudinal partial dielectric permittivity of particles of kind β, $\mathbf{k}_{\parallel}$ is the projection of the surface-wave vector on to the plasma boundary, e is the charge on the electron, m_β is the mass of particles of kind β, $J_m(a_j)$ is the Bessel function. Introducing the perturbations of the charged-particle density at the plasma boundary

$$\rho_\beta(z=0) = \Delta\varepsilon_\beta(\omega) \sum_{n=-\infty}^{\infty} \Phi(z=0, \omega + n\Omega, \mathbf{k}_{\parallel}) A_n$$

we can reduce Eq. (2) to a system which is identical with that studied in [1] except that $\Delta\varepsilon_\beta(\omega)$ is replaced by $\delta\varepsilon_\beta(\omega, \mathbf{k})$. We can therefore use the method of solution developed in [1].

We limit the analysis to the two most interesting types of field (1). For weak modulation $E_1 = E_2 = 1/2\alpha E_0$ and $\alpha \ll 1$. For strong modulation the field becomes

$$\mathbf{E}(t) = \mathbf{E}_0 \sin \omega_0 t + \mathbf{E}_1 \sin \omega_1 t \tag{3}$$

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and the amplitudes E_0 and E_1 can be comparable.

The maximum value of the growth rate for weak modulation at a frequency $2\omega_{Le}/\sqrt{1+\varepsilon_0}$ is

$$\gamma_{max} = 0.7\alpha\omega_{Li}\left[\frac{m_e}{m_i(1+\varepsilon_0)}\right]^{1/2} \tag{4}$$

where $\omega_{L\beta}$ is the plasma frequency for particles of kind β.

In the strongly modulated field (3) the maximum instability growth rate is equal to

$$\gamma_{max} = \frac{\omega_{Li}}{2}\sqrt{\frac{m_e}{m_i(1+\varepsilon_0)}}\sum_{m=-\infty}^{\infty}\frac{J_m(a_0)J_m(a_1)J_{m+1}(a_0)J_{m+1}(a_1)}{(2m+1)^2} \tag{5}$$

If it is not necessary to assume a sharp boundary in order to get the maximum instability growth rate, a calculation of the threshold modulation depth allowing for dissipative effects must take into account the spread in the boundary. We therefore consider the case of a nonuniform plasma which is separated from the surrounding medium by a transition layer of thickness a in which the density varies much more rapidly ($k_\| a \ll 1$) than in the main body of the plasma $(k_\|^{-1}\partial \ln n_0/\partial z|_{z>a} \ll 1$. The wave spectrum and the stability of such a plasma in a strong hf field have been studied in [3]. The presence of a nonuniform transition layer produces [4] dissipation of the surface modes because of the increase of the longitudinal field which occurs when the frequency of the surface modes approaches the local plasma frequency. Knowing the attenuation of the surface waves, we can determine the threshold value of the modulation depth α^* at the second harmonic

$$\alpha^* = 2.3\frac{m_i}{m_e}\left\{\frac{\varepsilon_0^2\pi}{1+\varepsilon_0}k_0\int_0^a dz\,\delta\,[\varepsilon(\omega_+, z)] + \frac{\nu_{ei}}{\sqrt{1+\varepsilon_0}\,\omega_{Le}}\right\}$$
$$\varepsilon(\omega, z) = 1 + \sum_\beta \delta\varepsilon_\beta(\omega, z) \tag{6}$$

Here ν_{ei} is the collision frequency of electrons with ions. The quantity k_0 which appears in Eq. (6) is found from the condition

$$\Omega = 2\omega_+\left\{1 - \frac{\omega_+^2}{8k_0^2c^2} + \frac{k_0}{2(1+\varepsilon_0)}\int_0^a \frac{dz}{\varepsilon(\omega_+, z)}[\varepsilon_0^2 - \varepsilon^2(\omega_+, z)] + \frac{1}{4k_0}\frac{\partial \ln\varepsilon(\omega_+, z)}{\partial z}\bigg|_{z\gg a}\right\}, \quad \omega_+ = \omega_{Le}(1+\varepsilon_0)^{-1/2} \tag{7}$$

The propagation angle in a plane parallel to the boundary and relative to the vector E_0 is given by

$$k_\| r_{E0} = 1.43 \tag{8}$$

It can be seen from Eq. (6) that in contrast to the volume-wave case [1] the threshold modulation depth at the second harmonic of the surface waves in the case of a finite transition layer is determined principally by collisionless absorption processes.

We now consider the modulation of the hf field by a weak low-frequency signal. This causes the excitation of low-frequency waves ($|\omega| \lessdot \omega_{Li}$). The resultant spectrum for a nonuniform plasma has been studied in [3]. For the threshold modulation depth we have

$$\alpha^* = 2.3\pi\frac{\varepsilon_0^2}{1+\varepsilon_0}k_0\int_0^a dz\,\delta\,[\varepsilon_1(z)], \quad \varepsilon_1(z) = 1 - (1+\varepsilon_0)\frac{\omega_{Li}^2(z)}{\omega_{Li}^2(a)}$$

Here k_0 is defined by Eq. (7) in which we have to put

$$\omega_+ = 0.9\omega_{Li}(a)(1+\varepsilon_0)^{-1/2}$$

The propagation angle at the threshold is defined by Eq. (8) and the maximum instability growth rate

$$\gamma_{max} = 0.24(1+\varepsilon_0)^{-1/2}\alpha\omega_{Li}$$

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